Graphical Introduction to The Derivative Function

Worksheet 3 is about $f(x)$ and $f'(x)$. This content is slightly ahead of us in lecture, so before you start the worksheet please read through this supplement (and ask your TA if you have questions).

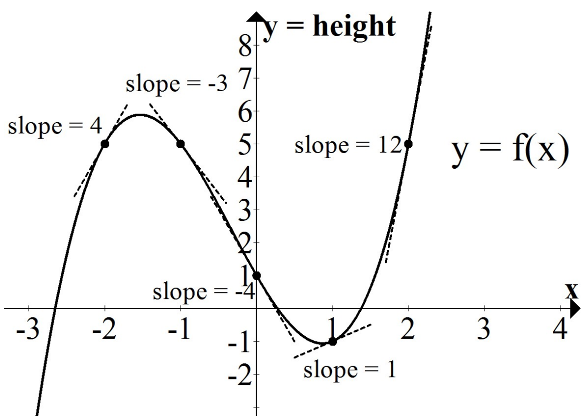
Let $y = f(x)$ be a given function. Recall, that $y = f(x)$ = 'the height of the function at x'.

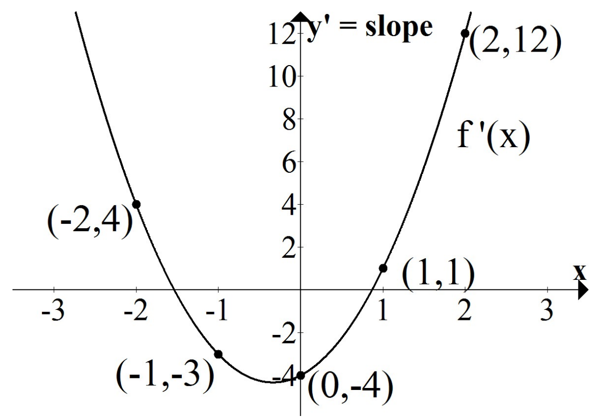
We define $y' = f'(x)$ = 'the slope of the tangent line to the function at x'.

In sections 2.7 and 2.8, we will study how to find this function exactly using limits. For worksheet 3, you do not need to know the material from 2.7 and 2.8, you only need to remember that
$y = f(x)$ = height and $y' = f'(x)$ = slope.

Example:
Below (on the left) is a graph of the function $y = f(x) = x^3 + x^2 - 4x + 1$.
If you plug in $x = 2$ you get $y = f(2) = (2)^3 + (2)^2 - 4(2) + 1 = 5$, so the height of the graph at $x = 2$ is 5. You can see this in the picture. I have also put dots at the heights corresponding to x = -2, -1, 0, 1, and 2. (all obtained by evaluating the function).

Now consider the slopes. We have drawn the tangent lines at $x = -2, -1, 0, 1,$ and 2. And we give the slope of each of these tangent lines (you will learn in chapter 3 how to find these values). On the left, we created a graph of all the slopes.





The goal of Worksheet 3 is to start to see the connections between these two graphs. Notice that the graphs do NOT look the same and they are giving very different information about the function. But there are connections between the graphs. Here are some of the connections I want you to think about:

ORIGINAL ($f(x)$)	DERIVATIVE ($f'(x)$)
$f(x)$ = height of original at x	$f'(x)$ = slope of original at x
increasing (uphill left-to-right)	positive (above x-axis)
decreasing (downhill left-to-right)	negative (below x-axis)
horizontal tangent	zero (crosses x-axis)

The facts above are fundamental to calculus and we will use them over and over and over again in applications. Eventually you will be able to use these even if you don't have a graph in front of you. These connections should be something you natural use without thinking about by the end of the term. At the end of this worksheet, you should have some strategies for the following two tasks:

1. Given the 'original' height graph, draw a rough sketch of the 'derived' slope graph.
2. Given the 'derived' slope graph, draw a very rough sketch of the general shape of the 'original' height graph.